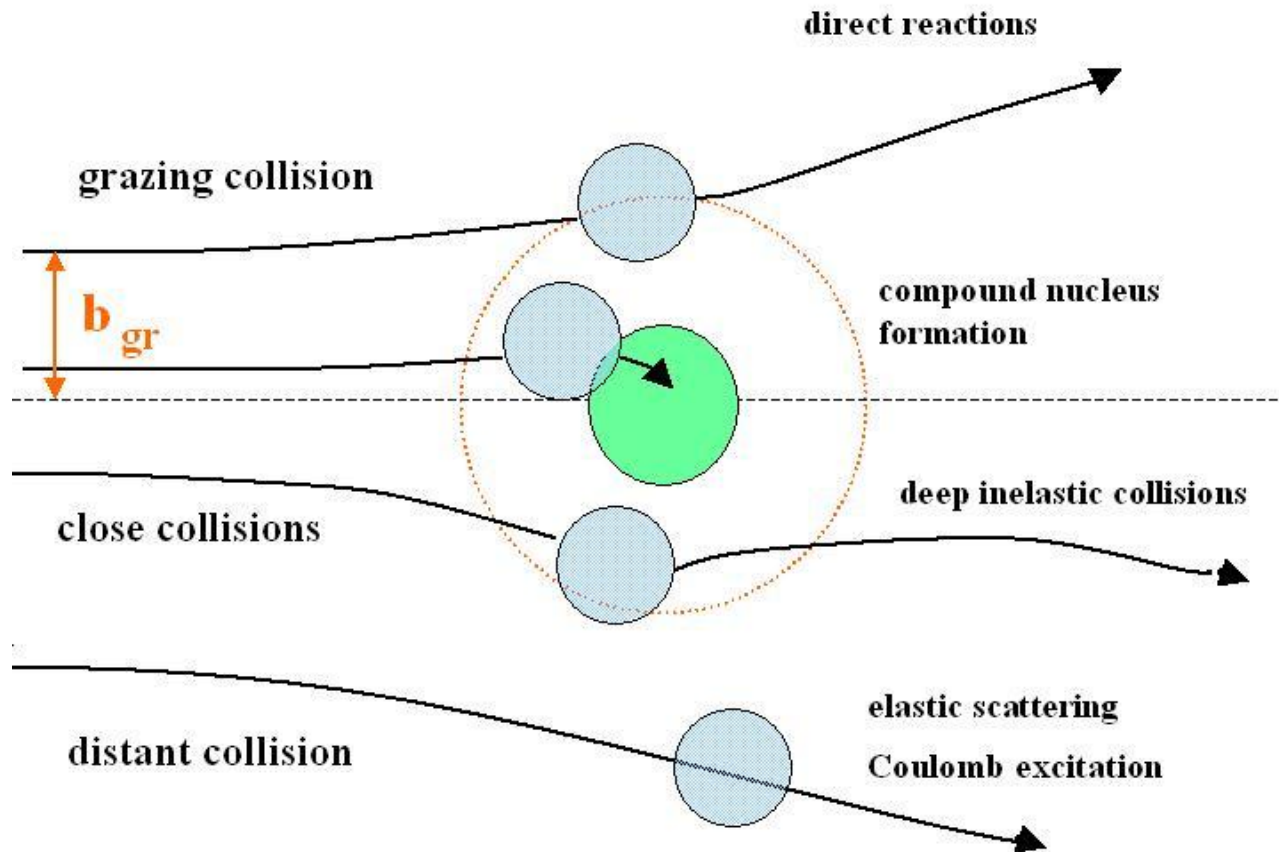
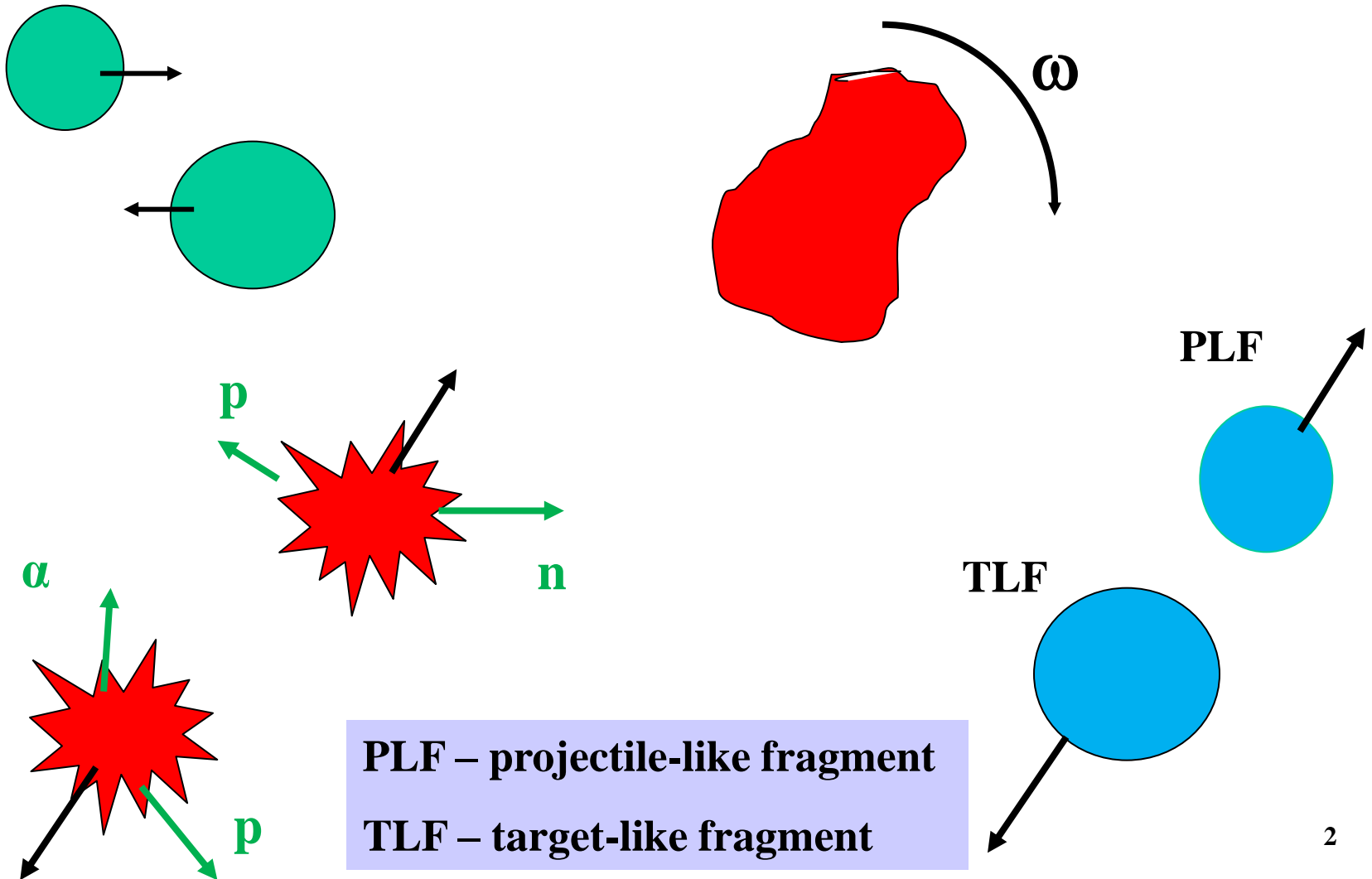


Classification of the heavy ion collisions at low energies



Damped collisions



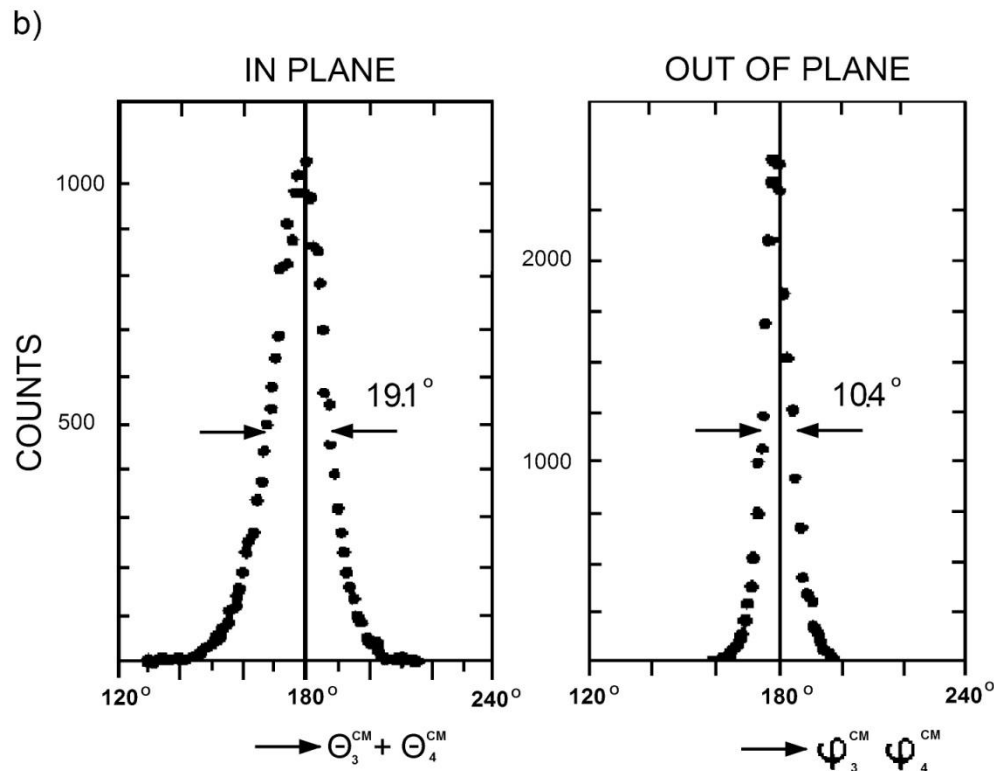
$$TKE = E_{kin;PLF}^{CM} + E_{kin;TLF}^{CM}$$

TKE – total kinetic energy in the exit channel

$$E_{LOSS} = E_{CM} - TKE$$

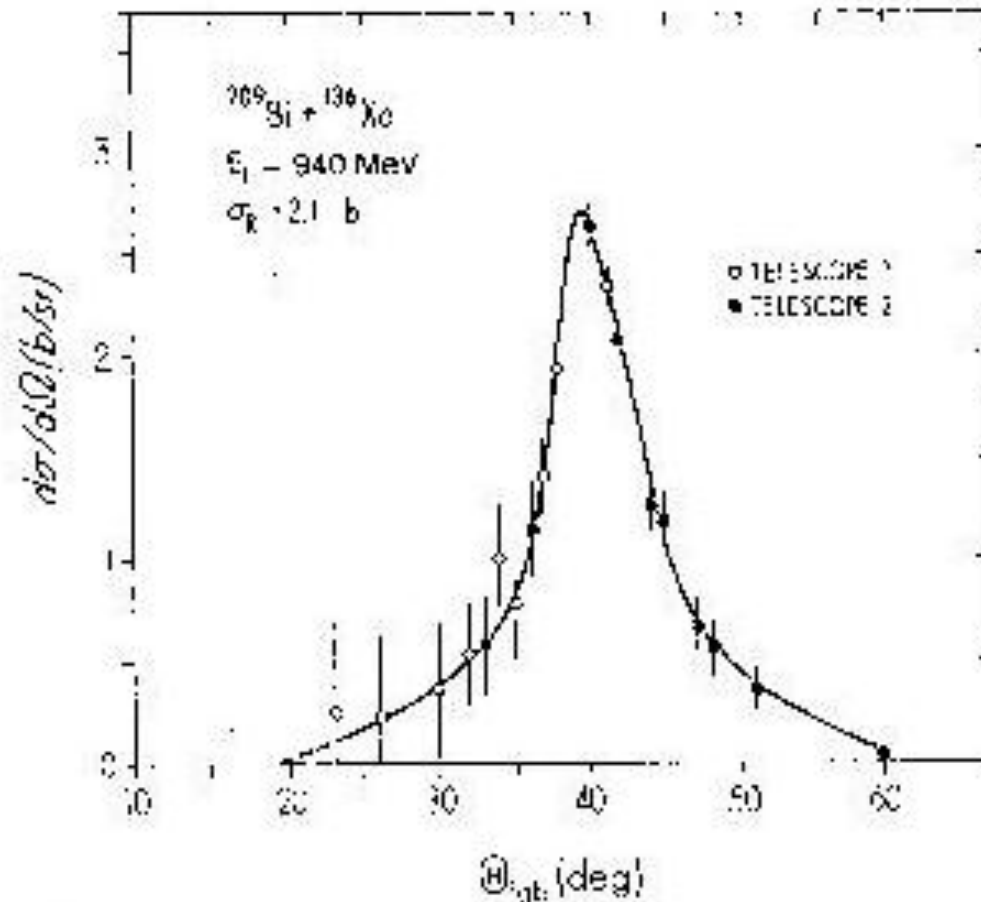
E_{LOSS} – total kinetic energy loss

- the reaction is binary in a sense that only two massive fragments are observed in the exit channels



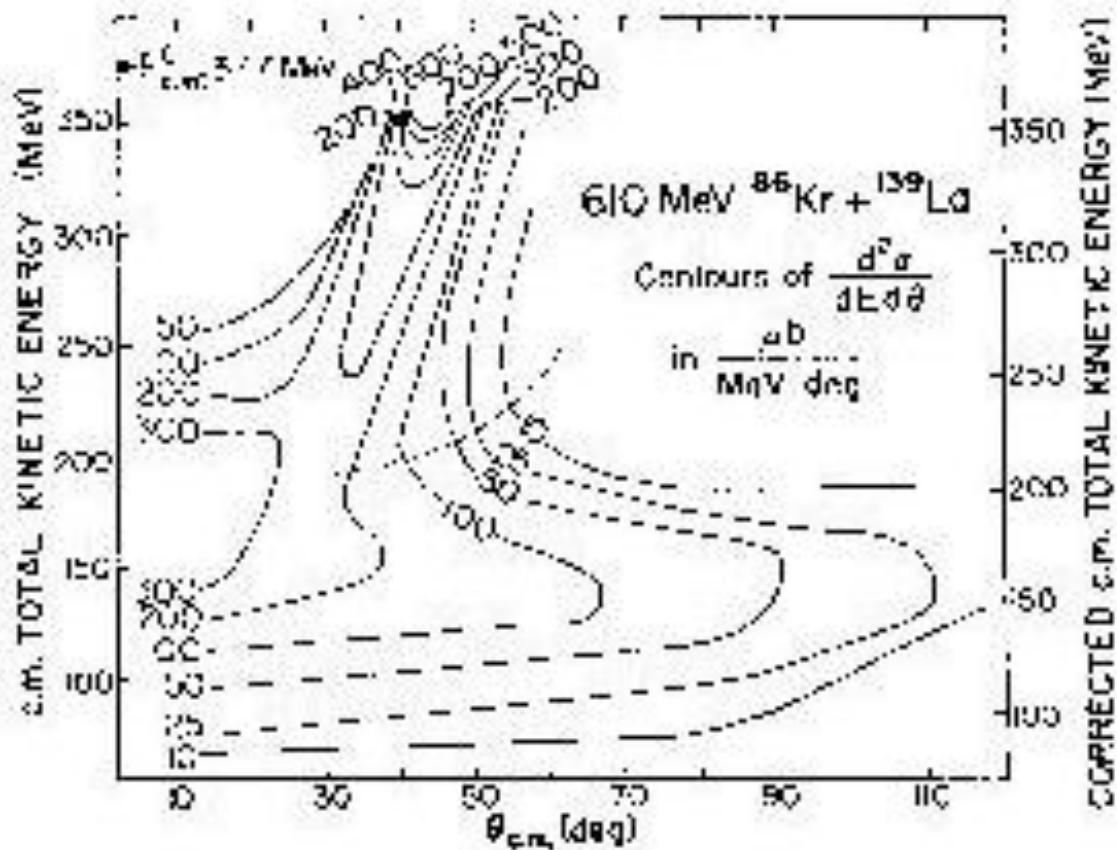
Measured deviations from colinearity for the in plane and out of plane distributions for all inelastic events in the $^{40}\text{Ar} + ^{58}\text{Ni}$ reaction at incident energy of 7 MeV/nucleon.

- angular distribution of the reaction cross section is sideways-peaked



Angular distribution of the Xe-like reaction products for $^{136}\text{Xe} + ^{209}\text{Bi}$ system at 6.9 MeV/nucleon. The curve is to guide the eye.

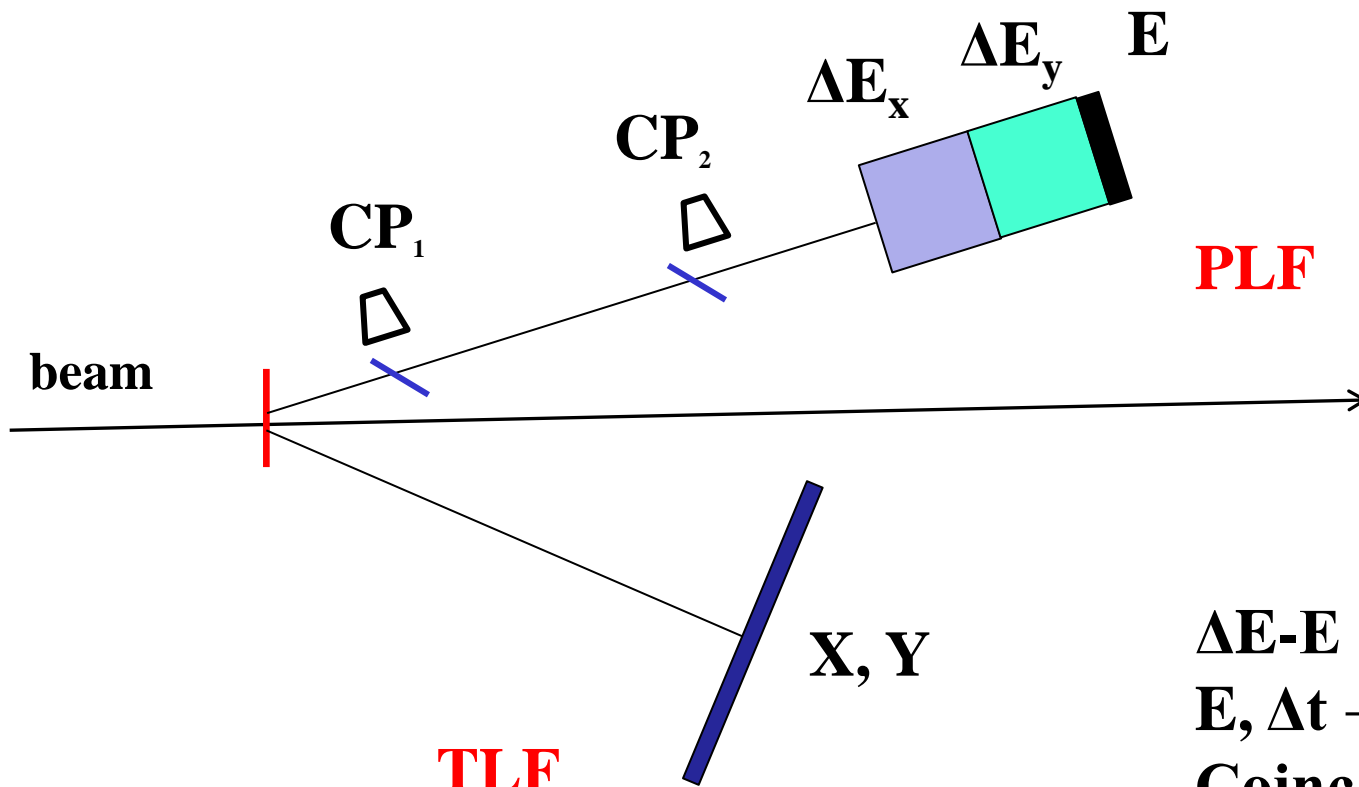
- the kinetic energy distribution of the final fragments is very broad.



Contours of the double differential cross section $(d\sigma/dEd\theta)_{CM}$ in $\mu\text{bMeV}^{-1} \text{deg}^{-1}$ for $^{86}\text{Kr} + ^{139}\text{La}$ reaction at 7.0 MeV/nucleon.

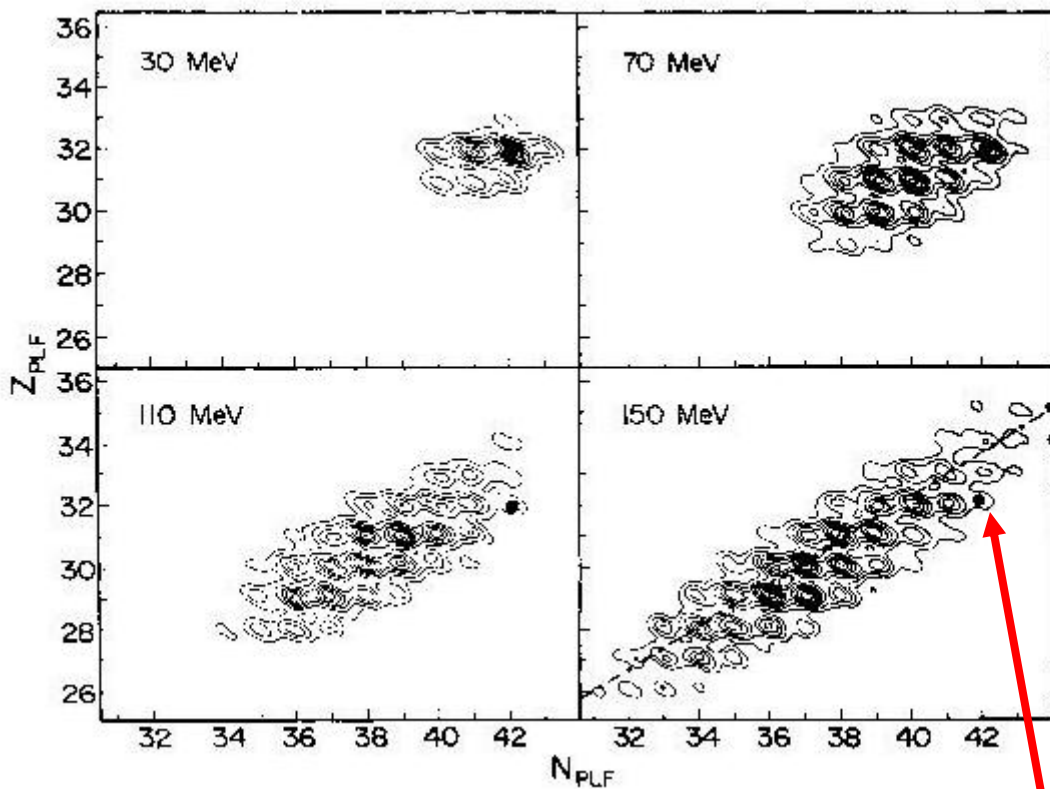
Experimental setup

$^{74}\text{Ge} + ^{165}\text{Ho}$,
 8.5 MeV/nucleon



$\Delta E - E \rightarrow Z(\text{sec})$
 $E, \Delta t \rightarrow A(\text{sec})$
 $\text{Coinc} \rightarrow A(\text{prim})$
 Statistical model
 $\rightarrow Z(\text{prim}), E^*$

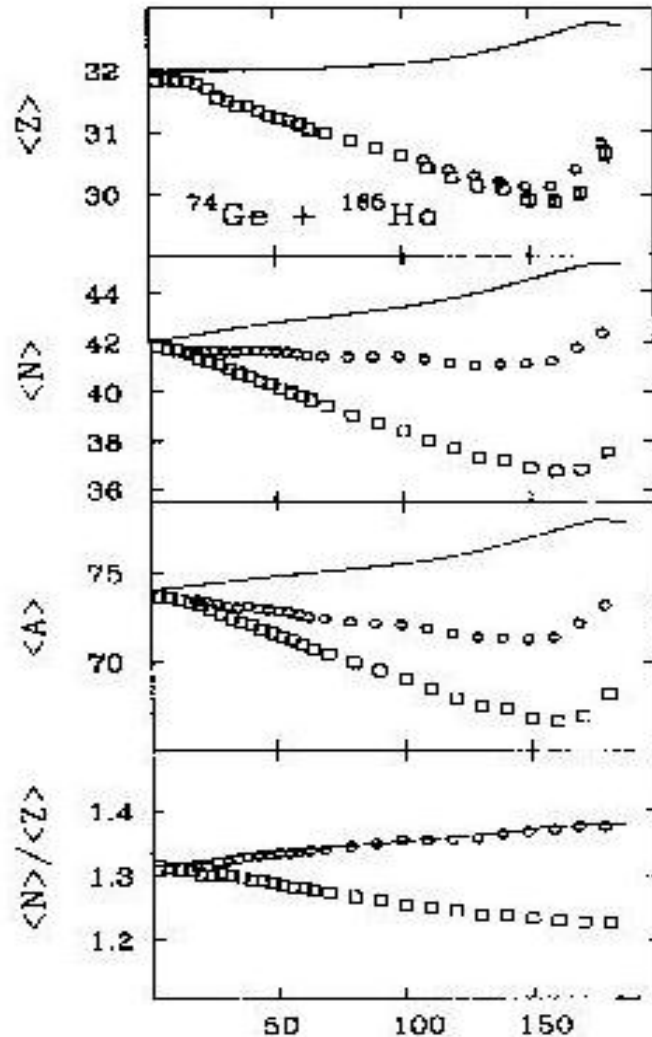
- the fragment mass and charge distributions are broad and located close to the mass and charge of the projectile and of the target nucleus, respectively.



Cross section contours in the N versus Z plane for the $^{74}\text{Ge} + ^{165}\text{Ho}$ reaction at four representative energy losses. Each bin width is $\pm 5\text{MeV}$ about the centroid. The dot-dashed line shows the line of maximum beta stability. Solid dot is ^{74}Ge .

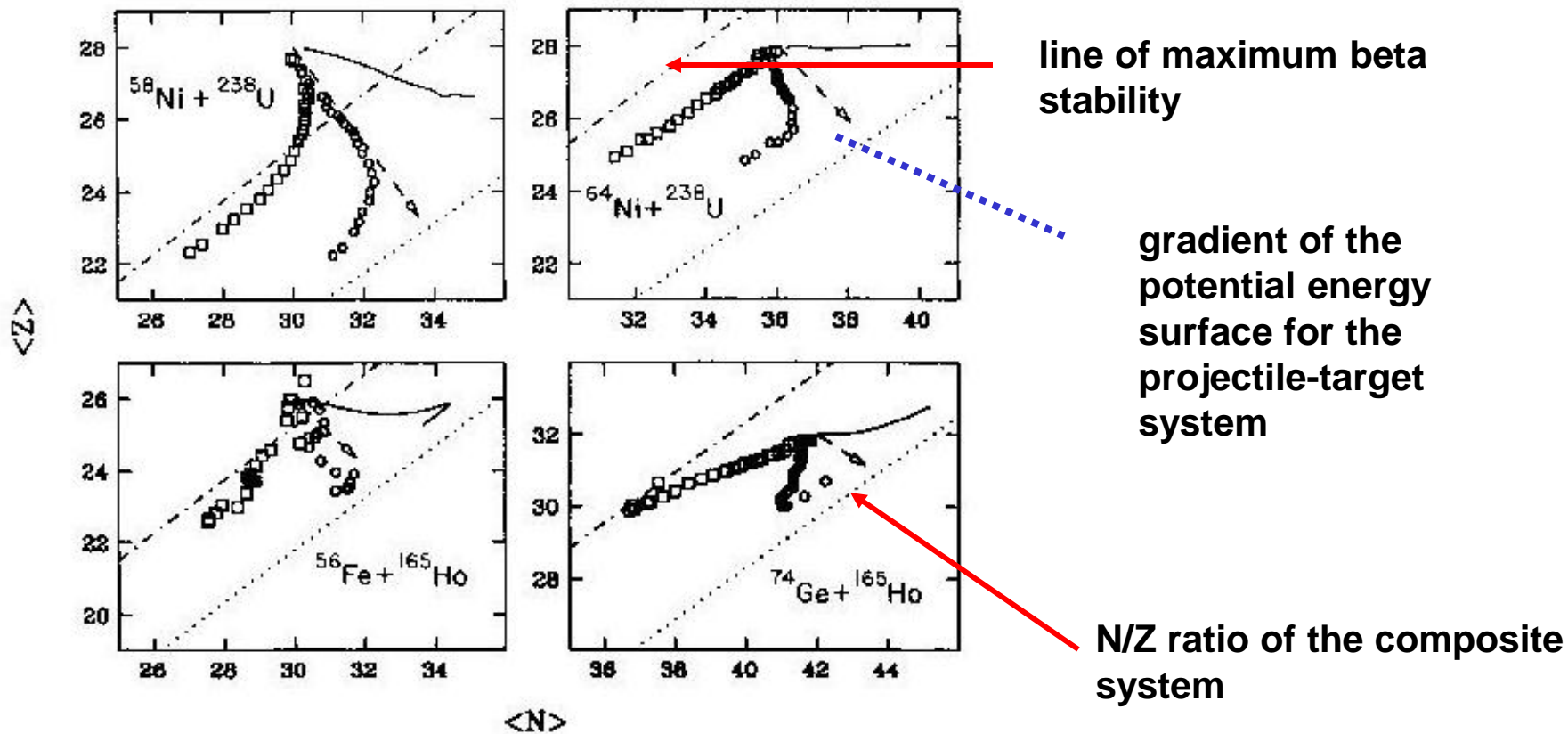
projectile

▪ strongly related to the evolution of the proton and neutron number centroids is the problem of the equilibration of the N/Z ratio.

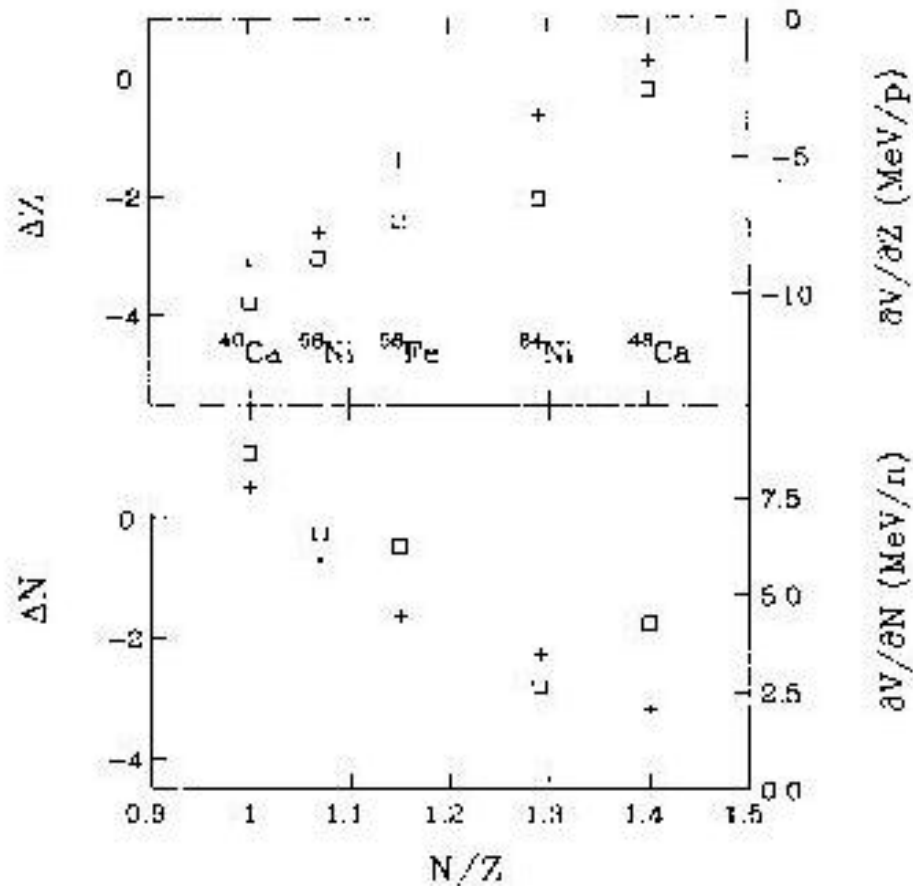


Centroids of the Z, N, and A distributions and the $\langle N \rangle / \langle Z \rangle$ ratio for PLF's as a function of energy loss

- Squares indicate measured post-evaporative values
- Circles give the primary values reconstructed from the kinematic coincident technique
- The solid line is the prediction of the nucleon exchange transport model



Evolution of net nucleon exchange as a function of energy loss for four systems. Measured distributions are indicated by squares, primary distributions by circles and theoretical predictions by solid lines.



Left scale:

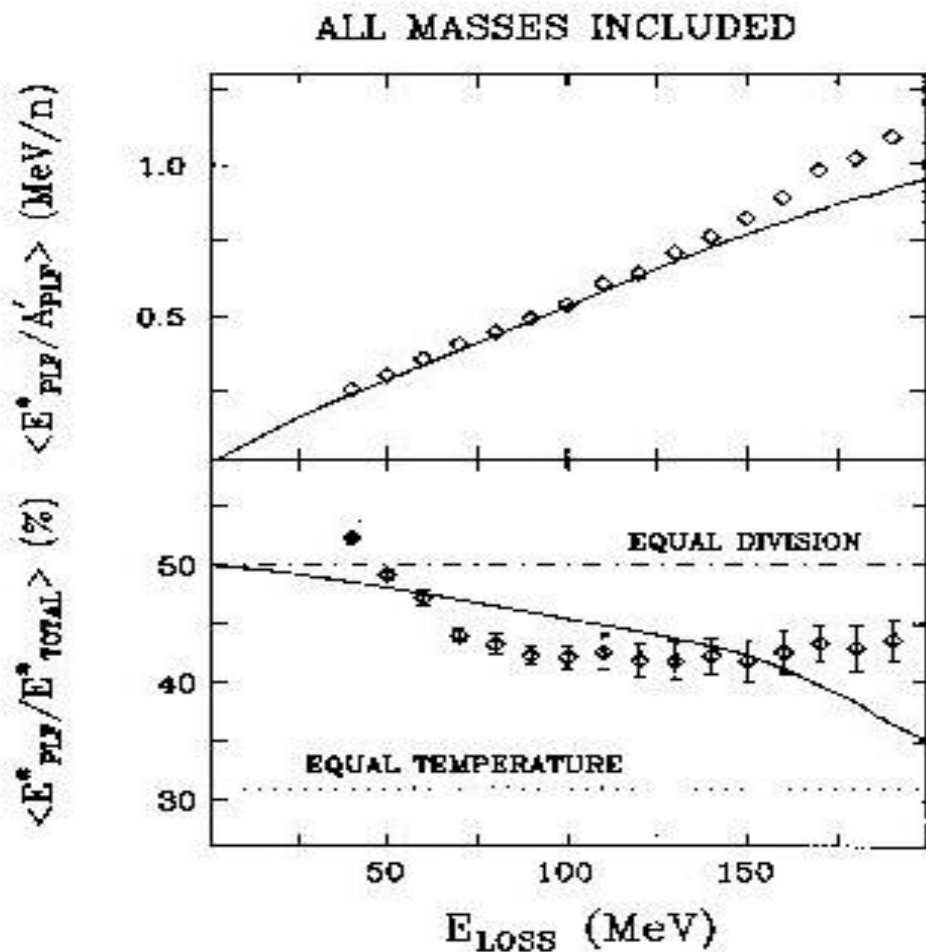
proton drift (upper panel) and neutron drift (lower panel) as a function of projectile N/Z value for $E/A = 8.5$ MeV $^{40, 48}\text{Ca}$, ^{56}Fe , and $^{58, 64}\text{Ni}$ ions incident on ^{238}U .

Right scale:

values of the gradient in the PES at the injection point (+) for considered systems.

$$E_{\text{LOSS}} = 100 \text{ MeV}$$

□ process of dissipation of kinetic energy of the entrance channel and its redistribution among the various degrees of freedom is of high significance for understanding of the damped reaction



$^{74}Ge + ^{165}Ho$

8.5 MeV/nucleon

$$E_{PLF}^* = E_{TLF}^*$$

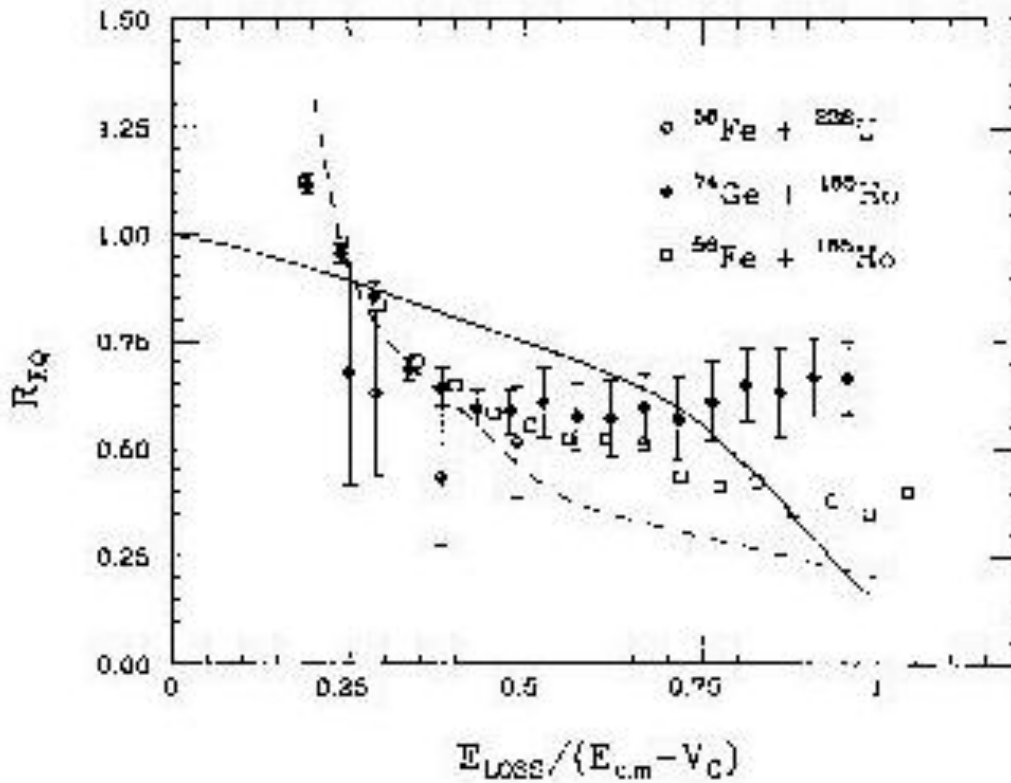
$$\frac{E_{PLF}^*}{E_{TOTAL}^*} = \frac{A_P}{A_P + A_T}$$

The factor R_{EQ} is defined as:

$$R_{EQ} = \frac{\left\langle \frac{E_{PLF}^*}{E_{total}^*} \right\rangle - \frac{A_P}{A_P + A_T}}{0,5 - \frac{A_P}{A_P + A_T}}$$

where A_P and A_T are the mass numbers of the projectile and target, respectively.

ALL MASSES INCLUDED

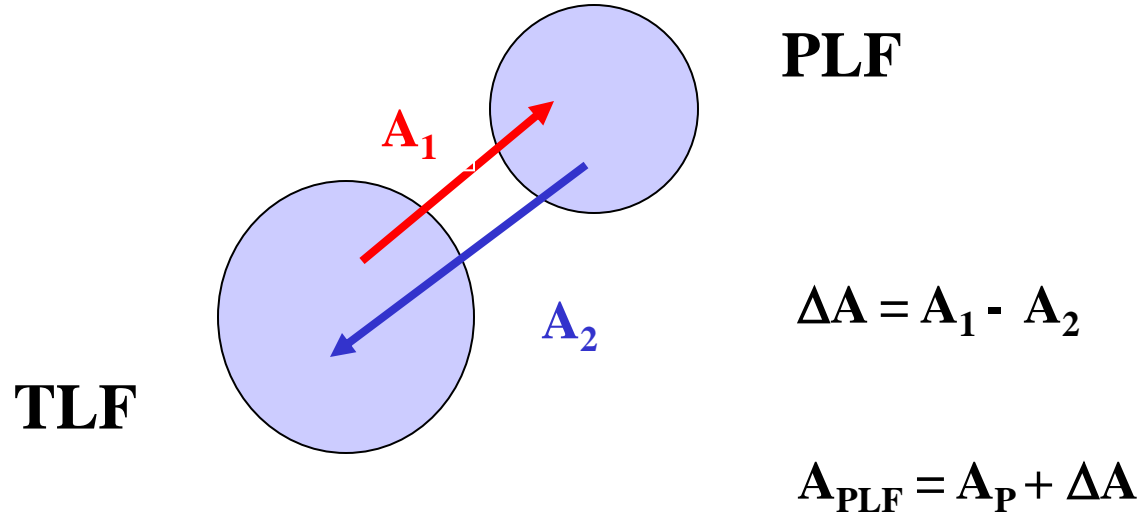


Parameter R_{EQ} as a function of energy loss divided by available excitation energy above the Coulomb barrier.

The data for the $^{56}\text{Fe} + ^{238}\text{U}$, $^{56}\text{Fe} + ^{165}\text{Ho}$, and $^{74}\text{Ge} + ^{165}\text{Ho}$ systems are presented.

The solid line is the prediction of the nucleon exchange transport model and the dashed line is based on the random neck rupture model .

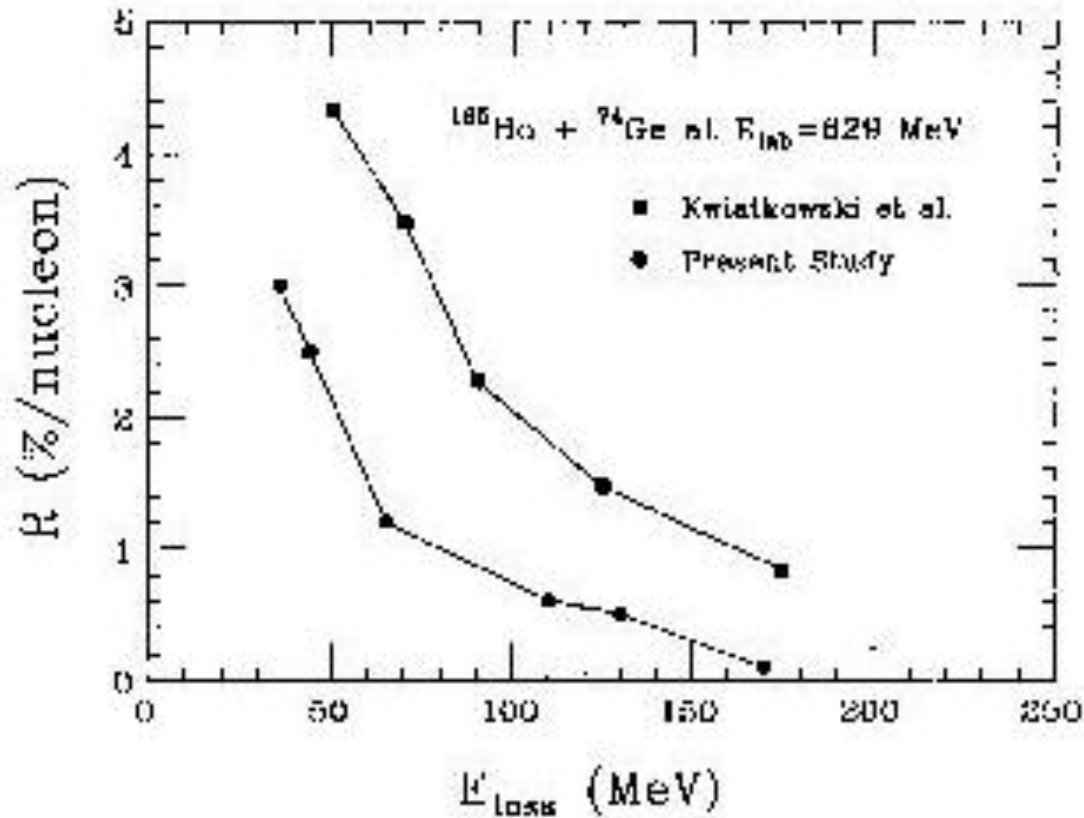
- some studies have also indicated that the partition of excitation energy is dependent on the net nucleon exchange .



The dependence between the E^*_{PLF} and the primary A_{PLF} was approximated by a linear function:

$$E^*_{PLF} = C + RE^*_{TOTAL} (A_{PLF} - A_0)$$

where C and R are E_{LOSS} dependent parameters. The A_{PLF} is the true primary mass and A_0 is the centroid of the primary mass distribution at a given E_{LOSS} .



Comparison of the strengths of the correlation between the excitation energy division and the net mass transfer in terms of the slope parameter R deduced in the straightforward analysis of (solid boxes) and in Toke et al.. analysis (solid dots).

Model description

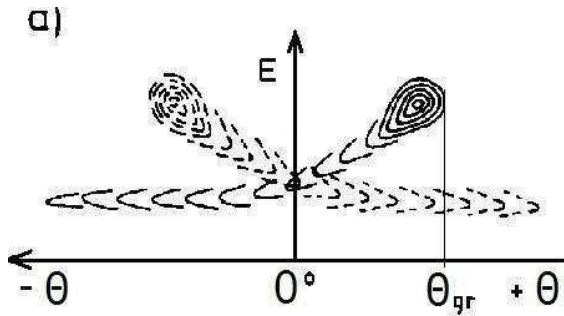
For the relative distance r the de Broglie wavelength is given by:

$$\hat{\lambda} = \left[\frac{2\mu^2}{\hbar} (E_{CM} - V(r)) \right]^{-1/2}$$

$V(r)$ is the interaction potential, μ is the reduced mass and E_{CM} is the energy of the system.

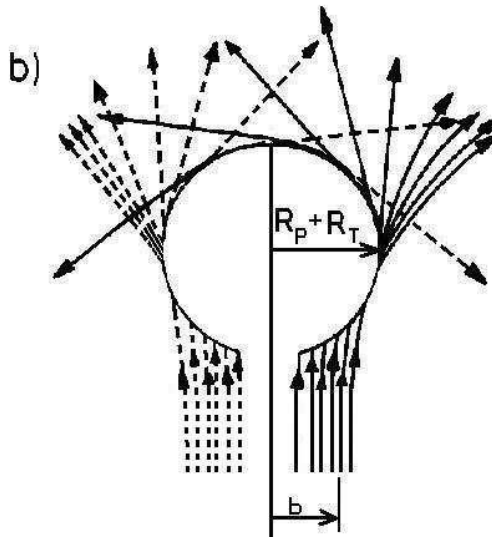
A more precise condition for classical behavior is given by:

$$| \text{grad } \hat{\lambda}(\vec{r}(t)) | \ll 1$$



Wilczyński plot:

(a) an energy versus scattering angle plot



(b) the figure illustrate the corresponding trajectories leading to the energy-angle correlation of part (a). The impact parameter is denoted by b .

A simple models include the transfer of charge and mass assuming that:

- **equations of motion in the entrance channel are integrated until the point of closest approach;**
- **transfer of neutrons and protons takes place only at R_{min} . At this point the relative velocity is corrected for the mass transfer effect;**
- **in the exit channel equations of motion are solved using a potential of the outgoing system.**

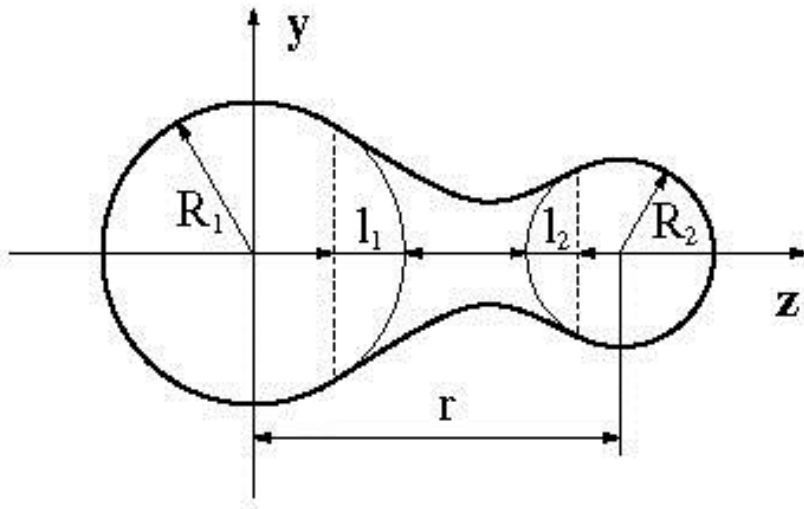
Model of Blocki:

$$\rho = \frac{r}{R_1 + R_2};$$

$$\lambda = \frac{l_1 + l_2}{R_1 + R_2};$$

$$\Delta = \frac{R_1 - R_2}{R_1 + R_2}.$$

where: ρ distance, λ neck, and asymmetry Δ variable.



$$\text{sphere 1: } y^2 = R_1^2 - z^2$$

$$\text{neck: } y^2 = a + bz + cz^2$$

$$\text{sphere 2: } y^2 = R_2^2 - (z - r)^2$$

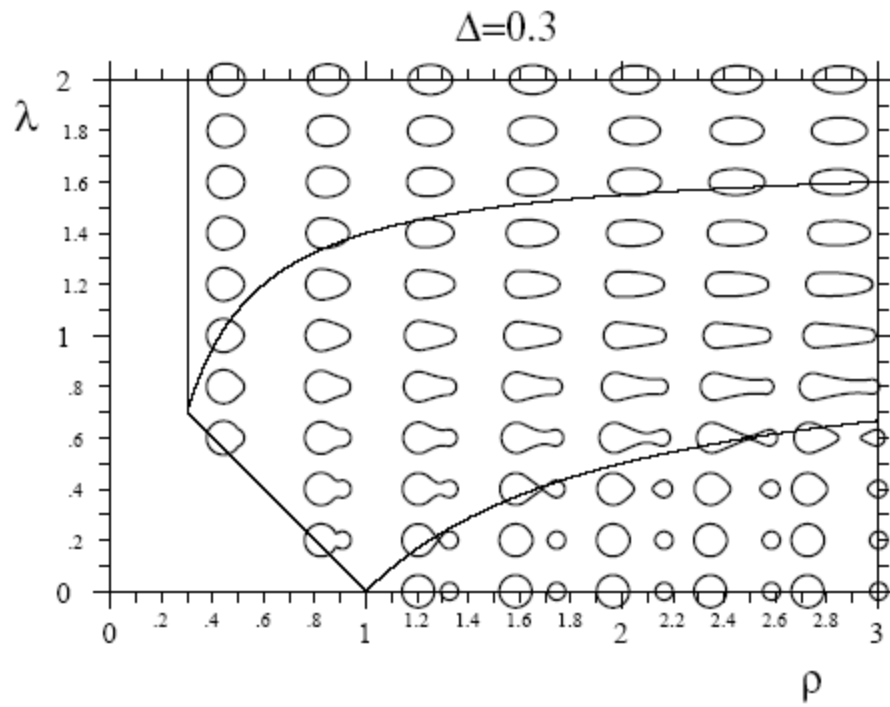


Fig. 1.4: The family of nuclear shapes for fixed asymmetry parameter $\Delta = 0.3$ as a function of the distance variable ρ and the neck variable λ .

$$\left[\frac{d}{dt} \frac{\partial}{\partial \dot{q}_i} - \frac{\partial}{\partial q_i} \right] L = - \frac{\partial}{\partial \dot{q}_i} F ;$$

where $L = T - V$ is the Lagrangian and F is the Rayleigh dissipation function. The kinetic energy is given as:

$$T = \frac{1}{2} \sum_{i,j=1}^3 M_{i,j} \dot{q}_i \dot{q}_j + \frac{1}{2} I_{rel} \omega_{rel}^2 + \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2$$

where $q_i = (\rho; \lambda; \Delta)$ is the set of shape parameters and rotation of the system is described with two spheres rotating with angular velocities ω_1 and ω_2 and the whole system rotating with angular velocity ω_{rel} . M_{ij} is a mass tensor calculated in the Werner-Wheeler approximation to irrotational ow. I_1 and I_2 are inertias of two spheres taken as rigid bodies and I_{rel} is the inertia of the relative rotation.

$$I_{rel} = I_{rigidbody}^{tot} - I_1 - I_2$$

In the more fundamental approach the nuclear part of the potential is calculated according to a double folding procedure developed by Krappe:

$$V_n = \frac{C_s}{8\pi^2 r_0^2 a^3} \iint \left(\frac{\sigma}{a} - 2 \right) \exp \left(\frac{-\sigma/a}{\sigma} \right) d^3 \vec{r} d^3 \vec{r}'$$

Where:

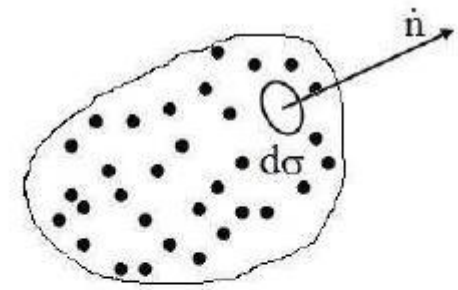
$$\sigma = \left| \vec{r} - \vec{r}' \right|,$$

$$C_s = a_s \left(1 - \kappa_s I^2 \right)$$

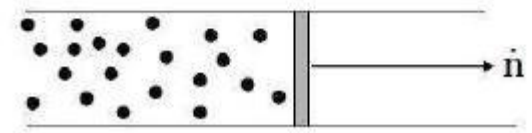
and parameters r_0 , a , a_s , and κ_s are taken from the fit done by Krappe.

Energy dissipation is given by:

$$\left(\frac{dE}{dt} \right)_{wall} = \rho \bar{v} \oint d\sigma (\dot{n} - D)^2$$



Fizyka

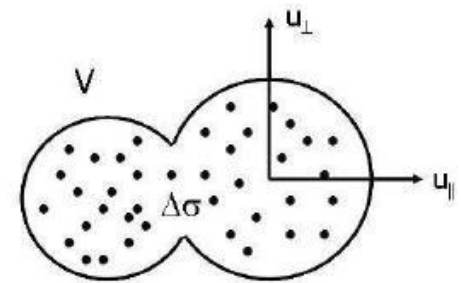


where ρ is the mass density of the nucleus, \bar{v} is the mean speed of nucleons in the nucleus, and \dot{n} is the normal velocity of an element $d\sigma$ of the nuclear surface. The quantity D is the overall drift velocity of the gas of nucleons.

$$\left(\frac{dE}{dt}\right)_{wall+window} = \rho \bar{v} \oint_{S_1} d\sigma (\dot{n} - D_1)^2 + \rho \bar{v} \oint_{S_2} d\sigma (\dot{n} - D_2)^2 +$$

$$+ \frac{1}{4} \rho \bar{v} (2u_r^2 + u_t^2) S_{\varpi} + \frac{16}{9} \frac{\rho \bar{v}}{S_{\varpi}} \dot{V}_1.$$

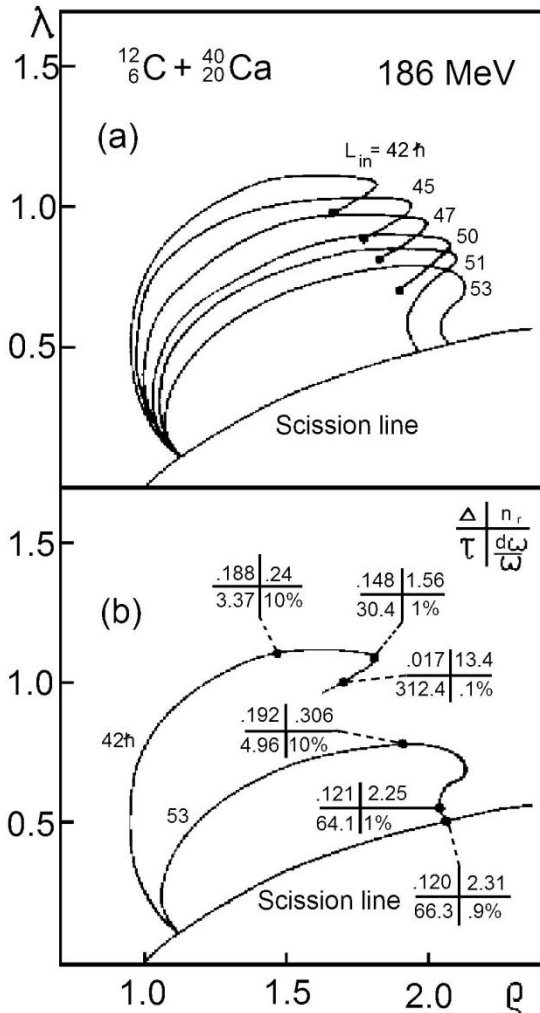
$$2F = f \left(\frac{dE}{dt}\right)_{wall} + (1-f) \left(\frac{dE}{dt}\right)_{wall+window}$$



Fizyka



with a form factor f going to 1 for sphere or spheroid like shapes and going to 0 at scission.



(a) Typical trajectories for the $^{12}\text{C} + ^{40}\text{Ca}$ reaction.

(b) For $L = 42\hbar$ and $L = 53\hbar$ the numbers written along the trajectories specify the asymmetry parameter Δ , the collision time τ (in units of 10^{-22} sec), the number of revolutions of the system n_r , and the maximum relative difference between ω_1 , ω_2 , $\omega_{rel.}$.

The Fokker - Planck transport equation:

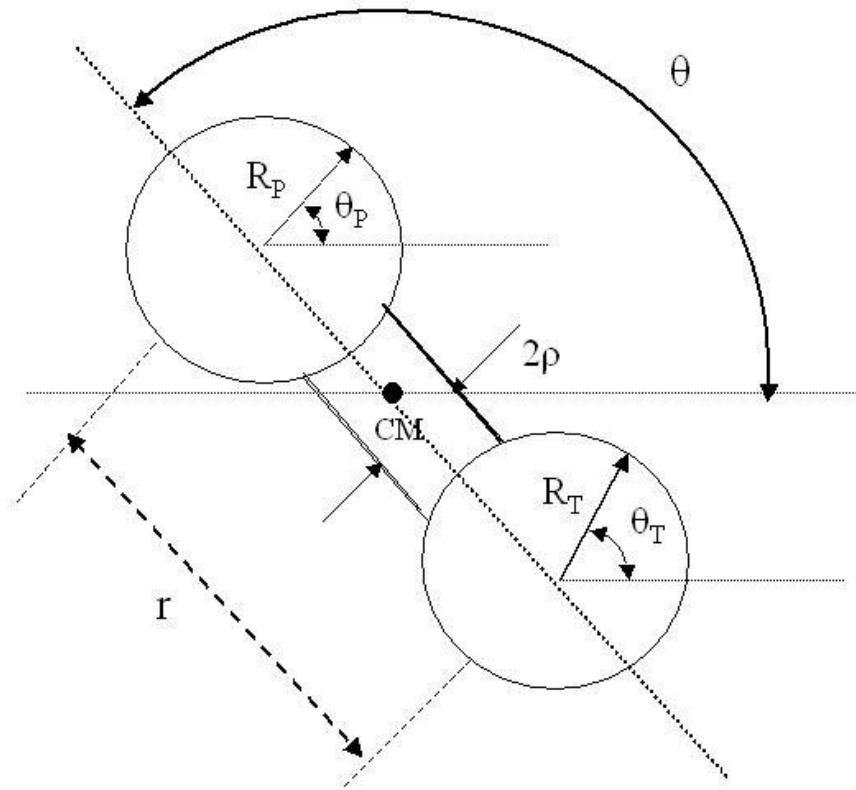
$$\left[\frac{\partial}{\partial t} + \dot{q} \nabla_q - (\nabla_q U) \nabla \dot{q} \right] P(q, \dot{q}, t) = - \sum_i \frac{\partial}{\partial q_i} [\nu_i(q, \dot{q}) P] + \sum_{i,j} \frac{\partial}{\partial q_i \partial q_j} [D_{i,j}(q, \dot{q}) P]$$

Here U is a potential, ν_i and D_{ij} are drift and diffusion coefficients, respectively. The left hand side describes the change of the probability distribution P due to the velocity \dot{q} and the force $-\nabla_q U$.

$$\frac{\partial}{\partial \bar{q}_i} L = \frac{\partial}{\partial \dot{q}_i} F$$

In the model proposed by Randrup the set of macroscopic variables is:

$$\{q_i\} = \{r, \theta, \theta_{PLF}, \theta_{TLF}, \rho, A_{PLF}, Z_{PLF}, T_{PLF}, T_{TLF}\}$$



Dinuclear shape coordinates assumed in the dynamical calculations

The Fokker-Plank equation reduces to:

$$\frac{d}{dt} P(N, Z, t) = \left[-\frac{\partial}{\partial N} v_N - \frac{\partial}{\partial Z} v_Z + \frac{\partial^2}{\partial N^2} D_{NN} + \frac{\partial^2}{\partial Z^2} D_{ZZ} \right] P(N, Z, t)$$

where v and D are drift and diffusion coefficients, respectively.